Fast, Accurate and Simple Approximate Analytic Formulas for Calculating the Parameters of Supported Coplanar Waveguides for (M)MIC's

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***Abstract*—Fast, accurate, and simple approximate analytic formulas are presented for calculating the quasi-static-TEM parameters of supported coplanar waveguide structures (SCPW's). These include, the open, covered as well as dielectric over-layed SCPW's. These formulas have been designed for use in (M)MIC's-CAD programs and are only valid whenever the supporting material is of lower permittivity. Comprehensive comparisons have been made by using a rigorous spectral-domain hybrid mode analysis. Accuracy is found to be better than 1 percent for most of the operating range of physical dimensions and available dielectric materials (ϵ_r = 1 to 20). Numerical results are also presented in order to investigate the properties of different SCPW structures.**

I. Introduction

Coplanar waveguides (CPW's) have received considerable attention due to several advantages offered over conventional microstrips especially for monolithic microwave integrated circuits (MMIC's) applications on GaAs substrates. Most of these efforts have been directed towards the obtaining of design parameters by either using available numerical methods, for example [1]–[12] or by deriving closed form expressions [13]–[20]. Some effort has also been made to investigate various considerations in the use of CPW's for (M)MIC's [21]–[27] even up to the millimeter-wave frequencies [21]. CPW is, often, considered to have free space above and below the dielectric substrate as shown in Fig. 1(a). This configuration has not been found suitable for GaAs (M)MIC's, where the GaAs substrate is typically thin and fragile, therefore it should be mounted on another material in order to increase its mechanical strength. One (already suggested and used) solution is to mount the GaAs substrate directly on a conducting backed ground plane (Fig. 1(b)). In this case, the ground plane will support the fragile substrate, thus increasing both the mechanical strength as well as the average power handling capacity of the structure. The resulting structure has been investigated thoroughly and some analytic formulas are also available, e.g., [5], [13], [14], [17], [18].

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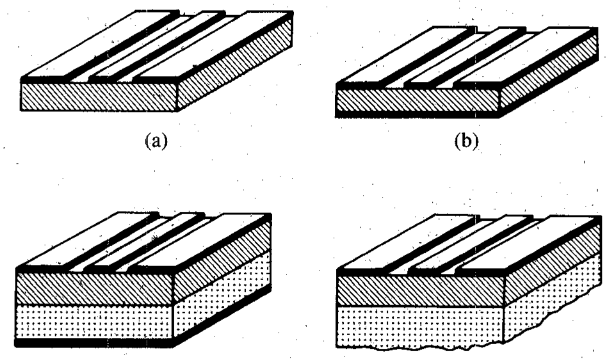

Fig. 1. Conventional and suggested supported CPW structures. (a) Conventional. (b) With direct backed ground plane. (c) With finite thickness supporting material and backed ground plane. (d) With infinitely thick supporting material.

It has been pointed out lately [21], [27] that there are some undesirable effects on the CPW behavior of the structure due to the presence of the microstrip mode in the presence of the backed ground plane. This mode can be easily suppressed by increasing the substrate thickness. However, this is not always possible, especially in (M)MIC's applications where semiconductor substrates are usually thin. A better solution is to mount the thin and fragile semiconductor substrate on a low-permittivity material such as quartz and then mount the entire assembly on a ground plane [21] (Fig. 1(c)). Another and even better solution is to grow a high quality GaAs layer on Si substrate and then mount the entire assembly on a ground plane [24] (Fig. 1(c)). In this case, the quartz or Si substrate will support the fragile GaAs substrate, thus improving both the mechanical strength as well as the average power handling capability of the structure. In both cases the presence of supporting dielectric material under the main substrate will enhance the effect of the microstrip mode [22]. Hence, the backed ground-plane should be placed far enough such that the consideration of this effect can be ignored (Fig. 1(d)). In order to assess in the computer-aided design of such structures, fast, accurate and

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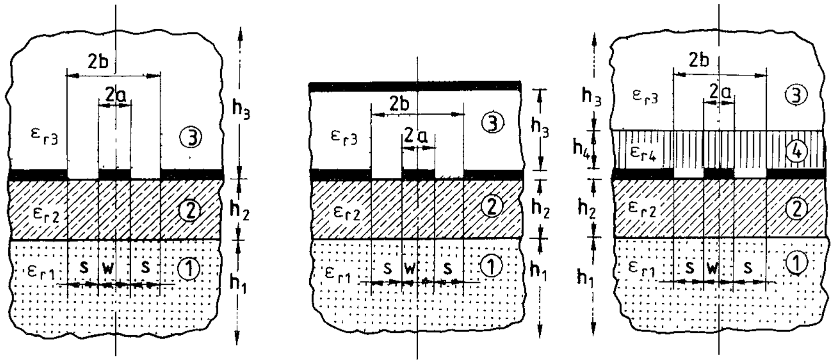

(a) (b) (c)

Fig. 2. Supported coplanar waveguide structures to be analyzed. (a) Open supported coplanar waveguide (SCPW1). (b) Covered supported coplanar waveguide (SCPW2). (c) Overlayed supported coplanar waveguide (SCPW3).

simple analytic formulas for the quasi-static-TEM parameters of these structures will be derived in this paper. Open, covered and dielectric overlayed SCPW's configurations (Fig. 2(a)–(c), respectively) are considered.

II. Analysis of Various Supported CPW's

Three supported structures will be considered here; these are:

1) The open supported CPW (Fig. 2(a)) with dielectric material (ϵ_{r1}) below the main dielectric substrate (ϵ_{r2}, h_2), while the space above the coplanar strips can be either filled with another dielectric (ϵ_{r3}) or air.

2) The covered supported CPW, which is similar to the open CPW except that there is a metallic top-cover placed above the coplanar strips at a distance h_3. The space between the main substrate and the top-cover can also be filled by either another dielectric (ϵ_{r3}) or air (Fig. 2(b)).

3) The third configuration is shown in Fig. 2(c). It consist of four layres: The main substrate (ϵ_{r2}, h_2), the supporting dielectric (ϵ_{r1}), the overlayed dielectric (ϵ_{r4}, h_4) as well as another dielectric material (ϵ_{r3}) or air. For the sake of brevity, these structures will be referred to in what follows as SCPW1, SCPW2, and SCPW3, respectively.

The approach used here is similar to that of [13]. It starts with finding an exact expression for the characteristic impedance Z_0^a (when replacing all dielectric materials by air) as well as assuming an approximate expression for the effective dielectric constant ϵ_{eff}. The second step is then to evaluate the error in the assumed expression by comparison with rigorous numerical results. If the differences are found to be very large, the assumed expression is then improved by curve fitting to the rigorous numerical results. These expressions may be written as

$$Z_0^a = \frac{1}{c_v C_t^a}, \tag{1}$$

where $c_v = 2.9979 \cdots 10^8$ m/s is the velocity of light in vacuum, and C_t^a is the total capacitance of the structure when replacing all dielectric materials by air, and,

$$\epsilon_{\text{eff}} = q_1 \epsilon_{r1} + q_2 \epsilon_{r2} + q_3 \epsilon_{r3} + q_4 \epsilon_{r4}. \tag{2}$$

The above expression is limited to a SPCW of, at maximum, four dielectric layers with q_1, q_2, q_3 and q_4 standing for the filling factors of dielectric regions 1, 2, 3, and 4, respectively. The value of the characteristic impedance in the presence of the dielectric materials is then calculated by using the following relation:

$$Z_0 = \frac{Z_0^a}{\sqrt{\epsilon_{\text{eff}}}}. \tag{3}$$

The expressions for the total capacitance C_t^a per unit length as well as the filling factors q_1, q_2, q_3 and q_4 can be obtained in terms of corresponding air filled basic capacitances per unit length.

In order to understand these basic capacitances, three new air filled CPW structures are considered in Fig. 3(a)–(c). These are corresponding to the original supported CPW's shown in Fig. 2(a)–(c), respectively and are obtained by replacing all dielectric interfaces in the original structures of Fig. 2 by magnetic walls. With reference to Fig. 3, four basic per unit length capacitances are defined as C_{I}^a, C_{II}^a, C_{III}^a and C_{IV}^a, representing the electric field in the regions I, II, III, and IV, respectively. The expressions for all these basic capacitances are available in literature [13]–[15]. These were obtained by conformal mapping technique and can be rewritten, in accordance with our physical dimensions, as follows:

$$C_i^a = 2\epsilon_0 \frac{K(k_i)}{K(k_i')} \quad (i = \text{I, II, III, and IV}) \tag{4}$$

where

$$k_{\text{I}} = \frac{w}{w + 2s}$$

$$k_{\text{II}} = \sinh\left(\frac{\pi w}{4h_2}\right) \Big/ \sinh\left(\frac{\pi(w + 2s)}{4h_2}\right),$$

$$k_{\text{III}} = \tanh\left(\frac{\pi w}{4h_3}\right) \Big/ \tanh\left(\frac{\pi(w + 2s)}{4h_3}\right),$$

$$k_{\text{IV}} = \sinh\left(\frac{\pi w}{4h_4}\right) \Big/ \sinh\left(\frac{\pi(w + 2s)}{4h_4}\right), \tag{5}$$

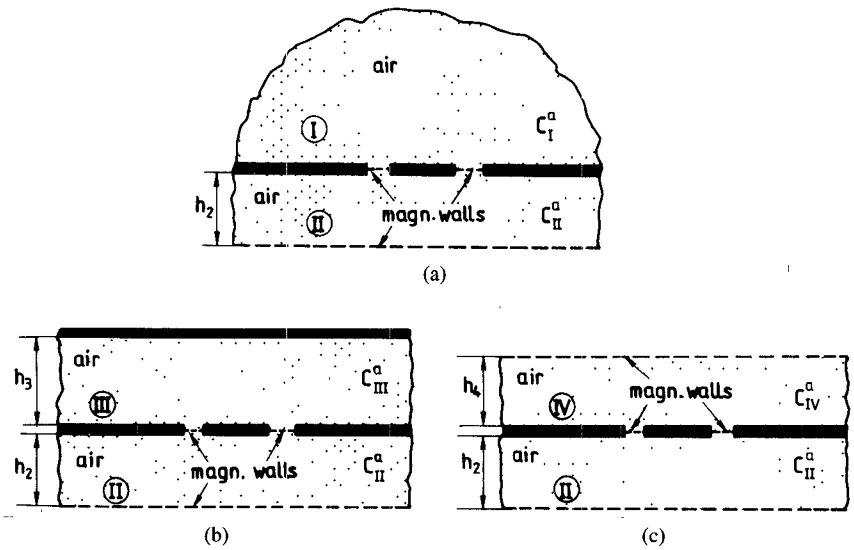

Fig. 3. Corresponding air filled CPW's. (a) SCPW1 correspondent. (b) SCPW2 correspondent. (c) SCPW3 correspondent.

with $K(k)$ and $K(k')$ as the complete elliptic integral of the first kind and its complement, and $k_i' = \sqrt{1 - k_i^2}$.

Accurate expressions for the ratio $K(k)/K(k')$ are available in [28]. These are given below:

$$\frac{K(k)}{K(k')} = \begin{cases} \frac{1}{\pi} \ln\,[2(1 + \sqrt{k})/(1 - \sqrt{k})] \\ \qquad 0.5 \le k^2 \le 1 \\ \pi/\ln\,[2(1 + \sqrt{k'})/(1 - \sqrt{k'})] \\ \qquad 0.0 \le k^2 \le 0.5. \end{cases} \tag{6}$$

The value of the capacitance C_t^a as well as the filling factors q_1 to q_4 can be written in terms of the above values of the basic capacitances. This will be made for each structure separately, in accordance to their practical importance, as follows.

A. Structure SCPW1

As shown in Fig. 2(a), the structure SCPW1 consists of three layers only. Therefore $q_4 = 0$. With reference to Figs. 2(a) and 3(a), the following exact values can be determined:

$$C_t^a = 2C_I^a,$$

$$q_3 = C_I^a/C_t^a = \tfrac{1}{2}. \tag{7}$$

In order to form an idea about the values of the filling factors q_1 and q_2 let us first investigate an assumed but a verified analytic formula for the effective dielectric constant of the conventional CPW shown in Fig. 1(a) where air exists above and below the main dielectric material. This formula, which has been first assumed by Veyres and Hanna [13] and then verified later by Ghione and Naldi [15], can be written as function of our basic air filled capacitances as follows:

$$\epsilon_{\text{eff}} = 1 + 0.5(\epsilon_r - 1)\frac{C_{II}^a}{C_I^a}. \tag{8}$$

But it can also be written as a function of the filling factor of the main substrate q_m as follows:

$$\epsilon_{\text{eff}} = (1 - q_m) + \epsilon_r q_m. \tag{9}$$

A simple comparison between (8) and (9), shows that the filling factor in this case is not a function of the relative dielectric constant of the main substrate and is only a function of the structure physical dimensions as follows:

$$q_m = 0.5\frac{C_{II}^a}{C_I^a} = \frac{C_{II}^a}{C_t^a}. \tag{10}$$

Since the assumption in (10) is verified for any air dielectric interface, then we can propose that it may also be valid for any other two dielectric interfaces (when replacing the air under the main dielectric substrate by another dielectric material). Therefore,

$$q_2 = q_m. \tag{11}$$

It should be pointed out that the conclusion derived from (10) is not generally correct, the filling factor should also be a function of the type of dielectric interface. Accordingly, the obtained value of the filling factor q_2 given by (10) and (11) should be considered as an approximate value and comparisons with rigorous numerical results is required to find out whether further improvement is necessary or not.

The value of the filling factor q_1 can then be determined from the values of q_3 and q_2 given by (7), (10), and (11) in the addition to the following well known relation,

$$q_1 + q_2 + q_3 = 1. \tag{12}$$

The following value for the filling factor q_1 can be obtained:

$$q_1 = (C_I^a - C_{II}^a)/C_t^a. \tag{13}$$

The values of Z_0 and ϵ_{eff} of SCPW1 can then be calculated by using (1) to (6) in association with (7), (10), (11), and (13).

B. Structure SCPW2

Reference is made to Figs. 2(b) and 3(b). This structure also consists of three layers, therefore $q_4 = 0$. The following exact relations can be obtained:

$$C_t^a = C_I^a + C_{III}^a,$$

$$q_3 = C_{III}^a / C_t^a. \tag{14}$$

The same assumed values for q_1 and q_2 of (10), (11), and (13) are used. The values of Z_0 and ϵ_{eff}, in this case, are calculated by using (1) to (6), (10), (11), and (13). It should be pointed out here that the value of C_t^a in this case would be different from that in the case of SCPW1 and which was given by (7).

C. Structure SCPW3

In this case, the reference is to Fig. 2(c) and 3(c), respectively. The following exact relation can be obtained

$$C_t^a = 2C_I^a. \tag{15}$$

The following additional approximate expressions can be also assumed in the same way which is followed in (8) to (13):

$$q_4 = C_{IV}^a / C_t^a,$$

$$q_3 = (C_I^a - C_{IV}^a)/C_t^a. \tag{16}$$

The same approximate values of (10), (11), and (13) for q_1 and q_2, are used. Z_0 and ϵ_{eff} are then calculated by (1) to (6), (10), (11), (13), (15), and (16).

The conventional CPW, shown in Fig. 1(a), can be considered as a limiting case for either SCPW1 (when both $\epsilon_{r1} = \epsilon_{r3} = 1$). In this case, the values of Z_0 and ϵ_{eff} converge to those of Veyres and Hanna [13].

Two assumptions have been made during the derivation. These are: The modeling of the two slots as magnetic walls as well as the assumed approximate values for q_1 and q_2 in case of SCPW1 and SCPW2 in addition to those of q_3 and q_4 in case of SCPW3.

The assumption of modeling the two slots as magnetic walls is always verified [19] in order to ensure proper behaviour of the structure as coplanar transmission line. The assumed values of the filling factors are logical, moreover they lead to the correct limits in the case of SCPW1 when $h_2 \rightarrow 0$, $h_2 \rightarrow \infty$ or $\epsilon_{r1} = \epsilon_{r2} = \epsilon_{r3} = 1$. However, comprehensive comparisons with results from a rigorous spectral domain hybrid mode analysis have had to be carried out. These have proven the validity of the assumed approximate values of q_1 and q_2 in case of SCPW1 and SCPW2 in addition to q_3 and q_4 in case of SCPW4 for a wide range of physical dimensions and dielectric constants. These have also shown that there is no need for further improvements for the assumed expressions. Some of these components and other results will be discussed in the following section.

III. Numerical Results

Numerical results will be presented in order to assess for the validity of the derived expressions as well as for investigating the properties of various SCPW configurations. Design charts will not be supplied, since the high speed computation formulas presented here can be programmed easily by the designers.

A. Results For the Assessment of the Validity of the Presented Expressions

The first group of numerical results are presented in order to assess for the validity of the presented formulas. Comprehensive comparisons with the results which are obtained by a rigorous spectral domain hybrid mode approach developed by Jansen [29], have shown that the accuracy of the derived formulas is better than 1 percent for most of the applicable range of physical dimensions used in (M)MIC's and available dielectric materials ($1 \leq \epsilon_r \leq 20$). The accuracy decreases as the spacing between the coplanar ground-planes ($2b = w + 2s$) increases, moreover, it is more sensitive to the increase in the slot width s than to a corresponding increase in the strip width w. Some of these comparisons are displayed in Tables I and II, respectively.

Table I shows comparisons with respect to the characteristic impedance of the structure SCPW1 with $h_2 = 200$ μm, $\epsilon_{r1} = \epsilon_{r3} = 1$ and $\epsilon_{r2} = 2.25$, 12.9, and 20.0, respectively. It should be pointed out here and again that in this case the presented expressions converge to those of Veyres and Hanna [13]. It should also be pointed out that Ghione and Naldi [19] have verified Veyres and Hanna's expressions but for a single dielectric material ($\epsilon_{r2} = 10$) by comparison with the upper and lower values which are obtained by spectral domain variational analysis. The main purpose of Table I is then to extend the validity of Veyres and Hanna's assumption (this paper's assumption) for a wider range of dielectric materials ($\epsilon_{r2} = 1$ up to $\epsilon_{r2} = 20$).

Table II shows a similar comparison for SCPW1 but with the presence of a supporting dielectric material and $\epsilon_{r3} = 1$. Three cases are displayed, these are: GaAs ($\epsilon_{r2} = 12.9$) supported by quartz ($\epsilon_{r1} = 3.78$), GaAs ($\epsilon_{r2} = 12.9$) supported by Alumina ($\epsilon_{r1} = 10$) as well as a hypothetical substrate ($\epsilon_{r2} = 20$) supported by Alumina ($\epsilon_{r1} = 10$).

It should be pointed out here that the numerical calculations using the spectral-domain hybrid mode approach have been carried out on a CONTROL DATA CYBER 76 computer, and the computing time for each point is found to be equal to 0.5 s. The number of spectral terms are truncated to 4000 and the dimensions of the enclosed shielding box are selected to be as in Fig. 4, in order to avoid the effect of both the top- and bottom-covers as well as the lateral side walls. The substrate thickness is chosen to be 200 μm and all presented calculations are made at a

TABLE I
COMPARISONS OF THE CHARACTERISTIC IMPEDANCES Z_0 (IN OHMS) OF THE CONVENTIONAL CPW WITH THE RIGOROUS SPECTRAL DOMAIN TECHNIQUE AND FOR DIFFERENT DIELECTRIC SUBSTRATE MATERIALS AT LOW FREQUENCIES (1 GHz)

		This Paper Expressions			Spectral Domain		
a/b	b/μm	$\epsilon_{r2} = 20$	$\epsilon_{r2} = 12.9$	$\epsilon_{r2} = 2.25$	$\epsilon_{r2} = 20$	$\epsilon_{r2} = 12.9$	$\epsilon_{r2} = 2.25$
0.2	50	54.49	67.95	140.75	55.76	68.28	141.40
	170	57.52	70.29	142.86	57.52	70.27	142.97
	230	59.00	72.13	144.43	59.02	71.95	143.95
	350	62.89	76.44	147.85	62.60	75.93	146.30
0.4	50	42.04	51.47	106.57	42.22	51.69	106.99
	170	43.88	53.60	108.47	43.86	53.56	108.32
	230	45.28	55.21	109.82	45.20	55.05	109.34
	350	48.46	58.82	112.62	48.24	58.44	111.41
0.6	50	33.32	40.80	84.45	33.48	40.99	84.83
	170	34.87	42.59	86.04	34.86	42.56	85.84
	230	35.99	43.87	87.12	35.93	43.76	86.68
	350	38.41	46.63	89.24	38.26	46.36	88.38
0.8	50	25.68	31.45	65.09	25.86	31.66	65.51
	170	26.81	32.75	66.03	26.80	32.71	66.03
	230	27.56	33.61	66.59	27.53	33.54	66.59

TABLE II
COMPARISONS OF THE CHARACTERISTIC IMPEDANCES Z_0 (IN OHMS) FOR THE OPEN SUPPORTED COPLANAR WAVEGUIDE SCPW1 WITH A RIGOROUS SPECTRAL DOMAIN TECHNIQUE AND FOR DIFFERENT DIELECTRIC INTERFACES AT LOW FREQUENCIES (1 GHz)

		This Paper Expressions			Spectral Domain		
s/μm	w/μm	$\epsilon_{r2} = 20$, $\epsilon_{r1} = 10$	$\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 3.78$	$\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 10$	$\epsilon_{r2} = 20$, $\epsilon_{r1} = 10$	$\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 3.78$	$\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 10$
20	20	45.51	55.96	55.91	45.85	56.37	56.33
	60	33.24	40.90	40.79	33.38	41.08	40.98
	120	27.68	34.11	33.90	27.12	34.16	33.96
	200	24.52	30.30	29.98	24.50	30.28	29.92
	800	19.01	23.73	22.60	18.83	23.72	22.47
60	20	61.82	76.09	75.84	62.01	76.32	76.09
	60	45.81	56.46	56.07	45.83	56.50	56.14
	120	37.73	46.62	46.01	37.67	46.56	46.00
	200	32.95	40.88	39.96	32.81	40.72	39.89
	800	24.41	30.99	28.69	24.05	30.56	28.43
100	20	70.56	86.97	86.36	70.60	87.03	86.48
	60	53.26	65.80	64.98	53.18	65.71	64.97
	120	44.09	54.65	53.92	43.90	54.44	53.42
	200	38.47	47.91	46.39	38.19	47.60	46.22
	800	27.98	35.76	32.61	27.48	35.16	32.25
200	20	83.77	103.81	101.75	83.24	103.18	101.48
	60	65.28	81.22	78.85	64.80	80.66	78.94
	120	54.87	68.62	65.81	54.31	67.95	65.44
	200	48.95	60.58	57.26	47.50	59.80	56.22
	800	34.42	44.60	39.47	33.66	43.65	38.95

frequency 1 GHz in order to avoid the effect of frequency dispersion. It should also be pointed out here that the comparisons with respect to the characteristic impedance would be sufficient and will automatically imply comparisons with respect to the effective dielectric constant. From the foregoing comparisons, it is concluded that no further improvements are required for the assumed values of q_1 and q_2 in case of SCPW1 in addition to those of q_3 and q_4 in the case of SCPW3.

B. Investigating the Properties of Various CPW Configurations

This second group of numerical results are performed in order to show the properties of the SCPW configurations, SCPW1, SCPW2 and SCPW3, respectively.

It can be observed from Table II ($\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 3.78$) and ($\epsilon_{r2} = 12.9$, $\epsilon_{r1} = 10.0$), respectively, that the design parameters of the SCPW1 seem to be less sensitive

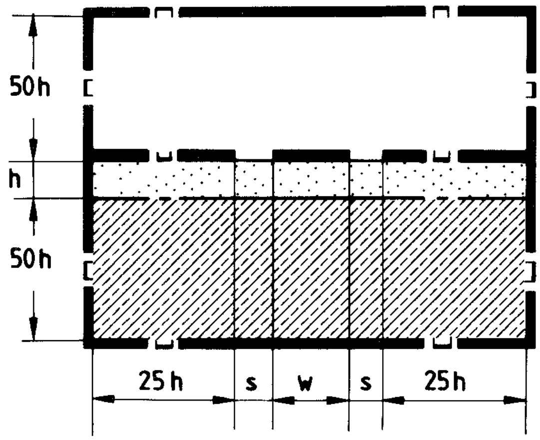

Fig. 4. Dimensions settings of the shielding box for the spectral domain calculations.

to changes in the permittivity of the supporting material. Fig. 5 shows the variation of the characteristic impedance of SCPW1 with the strip width $w = 20$ μm to 800 μm and for two values of the slot width $s = 20$ μm and 200 μm for a SCPW1 whose main substrate is GaAs ($h_2 = 200$ μm and $\epsilon_{r2} = 12.9$). The supporting dielectric material is also chosen to be either air ($\epsilon_{r1} = 1$), quartz ($\epsilon_{r1} = 3.78$), or alumina ($\epsilon_{r1} = 10.0$). It can be seen from Fig. 5 that the characteristic impedance of such a structure is less sensitive to changes in the permittivity of the supporting material for relatively small values of the ratio w/h_2 and s/h_2. However, the characteristic impedance becomes more sensitive for larger values of either the ratios w/h_2 and/or s/h_2. Moreover, the effect is found to be higher for the large values of the ratio s/h_2 than for a corresponding value for the ratio w/h_2.

Many references have investigated the effect of the presence of an overlayed dielectric layer on the characteristics of CPW's and hence there is no need to present more data here. In general, it has been observed that the presence of the overlayed dielectric material increases the value of the effective dielectric constant while it decreases the value of the characteristic impedance. The value of ϵ_{eff} also increases as the thickness of the overlayed material increases and approaches the double of its value for SCPW1 as the thickness approaches that of the main substrate (i.e., $h_4 = h_2$). The value of Z_0 decreases with the increase of the thickness of the overlayed material until approaching 0.7 of its value for SCPW1 as $h_4 = h_2$. It has also been observed that the impedance curves are overlapping, therefore, the same values for the characteristic impedance can be obtained for different sets of values of the physical dimensions of the structure SCPW3. This is due to the combined effect of the physical dimensions of the structure on the value of its characteristic impedance. It should be pointed out that the accuracy of the presented formulas for SCPW3 will deteriorate to 1.4 percent, this is of course due to the fact that, in this case, we used twice the approximation which is used in the case of SCPW1.

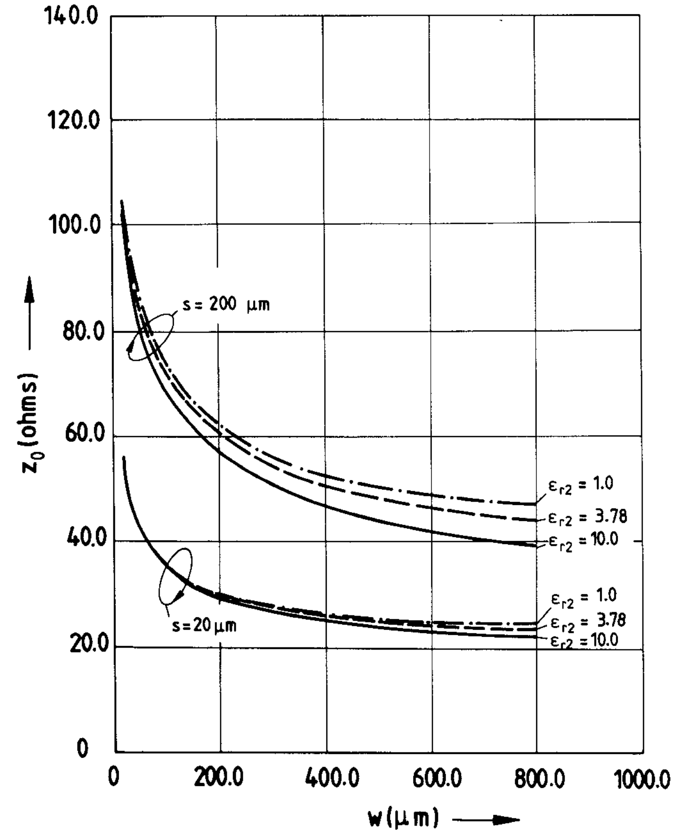

Fig. 5. Effect of the presence of different supporting materials on the characteristic impedance of SCPW1.

The effect of the presence of the top-cover as shown in SCPW2 has also been investigated. It has been observed that the decrease of the height of the top-cover h_3 will decrease both the values of the characteristic impedance and the effective dielectric constant of SCPW2. The effect of the presence of the top-cover decreases as the cover height h_3 increases until reaching a height at which this effect can be ignored. This is usually called the critical shield height and is defined as the height above which the effect of the presence of the top-cover can be ignored. A reasonable theoretical definition for a theoretical critical shield height to ground plane separation h_{c3}/b can be found as the ratio corresponding to the height of the top-cover h_{c3} above which the absolute difference between the characteristic impedance of SCPW1 and SCPW2 is less than 0.1 percent. The variation of this ratio as function of the strip width $w = 10$ μm to 400 μm and slot width $s = 20$ μm, 50 μm and 200 μm, is shown in Fig. 6. The main substrate is again chosen as GaAs ($\epsilon_{r2} = 12.9$ and $h_2 = 200$ μm), the supporting material is quartz ($\epsilon_{r1} = 3.78$) and the space between the GaAs substrate and the top-cover is filled with air. The presented formulas can either be used to consider the effect of the top-cover on the performance of a given covered SPCW circuit or to design SCPW circuits in which the effect of the top cover can be ignored.

Although the derived expressions are only rigorously

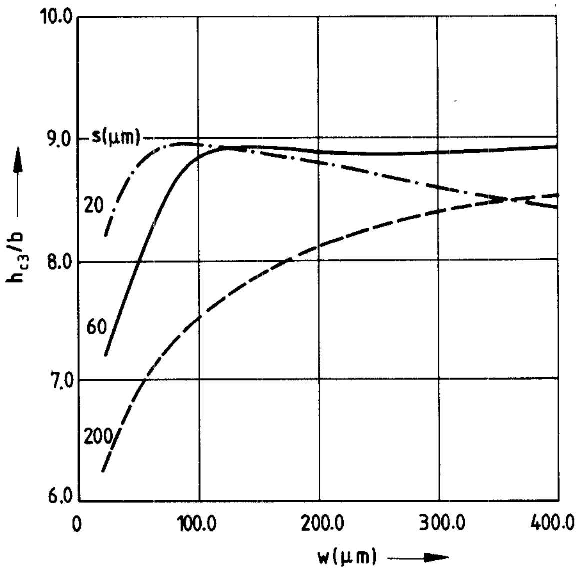

Fig. 6. Variation of the critical shield height to ground plane separation ratio h_{c3}/b as a function of the strip width w and for various slot widths. See text for parameters.

TABLE III
DISPERSION IN Z_0 AND ϵ_{eff} OF SUPPORTED CPW'S CALCULATED WITH A SPECTRAL DOMAIN TECHNIQUE [29]

	$s = 200\ \mu$m and $w = 120\ \mu$m				$s = 20\ \mu$m and $w = 120\ \mu$m			
f/GHz	Z_0/Ω	$\Delta Z_0\%$	ϵ_{eff}	$\Delta\epsilon_{eff}\%$	Z_0/Ω	$\Delta Z_0\%$	ϵ_{eff}	$\Delta\epsilon_{eff}\%$
1	67.97	—	6.2932	—	34.16	—	6.8431	—
5	68.09	0.18	6.3014	0.05	34.17	0.03	6.8448	0.02
10	68.32	0.51	6.3210	0.17	34.19	0.09	6.8490	0.09
15	68.57	0.88	6.3481	0.35	34.20	0.12	6.8552	0.18
20	68.81	1.24	6.3816	0.56	34.22	0.18	6.8630	0.29
25	69.03	1.56	6.4208	0.80	34.23	0.20	6.8724	0.43
30	69.22	1.84	6.4651	1.08	34.24	0.23	6.8833	0.59
35	69.36	2.05	6.5141	1.39	34.24	0.23	6.8955	0.77
40	69.44	2.16	6.5675	1.73	34.23	0.20	6.9091	0.96
45	69.46	2.19	6.6248	2.08	34.22	0.18	6.9239	1.18
50	—	—	—	—	34.20	0.12	6.9400	1.42
55	—	—	—	—	34.16	0.00	6.9573	1.67
60	—	—	—	—	34.12	−0.12	6.9758	1.94
65	—	—	—	—	34.06	−0.29	6.9955	2.23

valid at zero frequency, our investigations by using the spectral-domain technique in addition to the earlier investigations of Jackson [21] have demonstrated that they can be used for the design of GaAs-(M)MIC's up to a frequency of 40 GHz. This is because the ground planes spacing of these structures when used for MMIC's applications are typically small and the electromagnetic field is closely connected to the two slots at all frequencies if the slot width is small and thus the dispersion of the quasi-static TEM parameters is relatively small. Data obtained by a rigorous spectral domain hybrid mode approach [29] are presented in Table III. Two single CPWs are considered, both are deployed on GaAs substrate ($\epsilon_r = 12.9$ and thickness $h_2 = 200\ \mu$m), while the whole structure is mounted over quartz dielectric material of $\epsilon_r = 3.78$. The first CPW (of main strip width $w = 120\ \mu$m and slot width $s_g = 200\ \mu$m) shows deviations at a frequency $f = 45$ GHz in the characteristic impedance and the effective dielectric constant by amounts of 2.19 and 2.08 percent, respectively. While, the second CPW which has the same value of strip width but with slot width $s_g = 20\ \mu$m, shows deviations in the characteristic impedance and the effective dielectric constant, at a frequency $f = 65$ GHz by amounts of -0.29 and 2.23 percent, respectively.

IV. CONCLUSION

Fast and accurate analytic formulas have been presented for calculating the quasi-static TEM design parameters of open, covered and overlayed supported coplanar waveguide (SCPW's) structures, including the conventional CPW with free space above and below the main substrate as a special case. Comprehensive comparisons between the results which are obtained by using the derived formulas on one hand, and those obtained by a rigorous spectral-domain hybrid mode analysis on the other hand, have shown an excellent accuracy of better than one percent for most of the practical ranges of physical dimensions. The formulas presented here are about 5000 times faster than the rigorous analysis, hence they are especially applicable in CAD of (M)MIC design.

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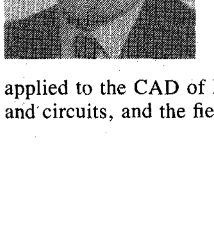

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